

## EULER STUDENT SAMPLE PROBLEMS

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### PROBLEM 1

Both the leftmost digit and the rightmost digit of a four-digit number  $N$  are equal to 1. When these digits are removed, the two-digit number thus obtained is  $N \div 21$ . Find  $N$ .

### PROBLEM 2

Find all three-digit even numbers  $N$  such that  $693 \times N$  is a perfect square, that is,  $693 \times N = k^2$  where  $k$  is an integer.

### PROBLEM 3

The parliament of the land of Achronia consists of two houses. The parliament was elected in 1995 for a period of four years beginning on Monday, January 1st, 1996, when the two houses had their first sessions. According to the rules, the meetings of the first house must occur every ten days for the duration of the term, and the meetings of the second house must occur every twelve days. For example, the second meetings of the first and the second houses were held on the 11th and 13th of January respectively. A new law can be passed only when both houses meet on a Monday. How many opportunities will the parliament members have to pass new laws during this four year term?

### PROBLEM 4

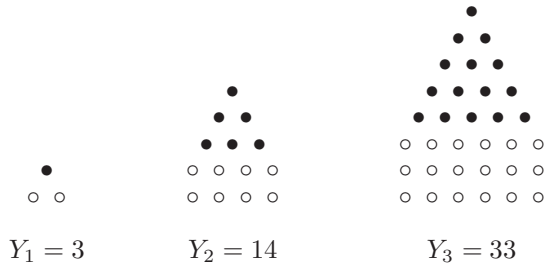
When a certain number  $N$  is divided by  $d$ , the remainder is 7. If the original number  $N$  is multiplied by 5 and then divided by  $d$ , the remainder is 10. Find  $d$ .

### PROBLEM 5

Find the sum of all positive integers not greater than 10000 that are divisible by either 3 or 11 but not by both of them.

**PROBLEM 6**

Consider the following pattern:



- (a) Find  $Y_5$ .
- (b) Find a rule for  $Y_n$ .
- (c) Find  $Y_{20}$ .

**PROBLEM 7**

- (a) Find the remainder when  $27^{1001}$  is divided by 13.
- (b) Find the remainder when  $38^{101}$  is divided by 13.
- (c) Hence show that  $70 \times 27^{1001} + 31 \times 38^{101}$  is divisible by 13.

**PROBLEM 8**

The “Good Book” publishing company invited representatives from a certain number of schools to its presentation where it planned to donate 3050 books to the schools so that each school would get the same number of books. But it turned out that eleven of the schools initially invited did not come to the presentation. However, the publishing company managed to divide the books equally among the schools that attended the ceremony. How many books did each of the schools get at the presentation?

**PROBLEM 9**

In a quadrilateral  $ABCD$ ,  $\angle A = 120^\circ$  and  $\angle B = 140^\circ$ . The bisectors of the angles  $C$  and  $D$  meet at the point  $Y$ . Find the size of the angle  $DYC$ .

**PROBLEM 10**

Wetlands High School has done it! It is the lucky school to win a trip to Disneyland for 2 girls, 2 boys and a teacher. The 5 lucky people are to be selected randomly from the school's computer records. If there are 100 boys, 80 girls and 20 teachers on the record, in how many ways can the group of 5 be selected?

**PROBLEM 11**

Ninety one five-digit numbers are written on a blackboard. Prove that one can find three numbers on the blackboard such that the sums of their digits are equal.

**PROBLEM 12**

Tom was floating down the river on a raft when, 1 km lower down, Michael took to the water in a rowing boat. Michael rowed downstream at his fastest pace. Then he turned around and rowed back, arriving at his starting point just as Tom drifted by. If Michael's rowing speed in still water is ten times the speed of the current in the river, what distance had Michael covered before he turned his boat around?